Simulating condensed matter with ultracold atoms: the Haldane model and the Peierls substitution

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Abstract

The Haldane model is a celebrated two-dimensional tight-binding model characterized by a quantum Hall effect due to the breaking of time-reversal symmetry, with zero net magnetic flux through the unit cell [1]. It is characterized by exotic quantum phases with different Chern numbers, depending on the value of the phase of the next-to-nearest tunneling amplitude. In its original formulation [1], the model is constructed by means of the so-called Peierls substitution (PS) [2,3], a popular approach that is widely employed in the literature to account for the effect of a vector gauge field A in the tight-binding description of electrons, as well as for ultracold atoms in optical lattices in the presence of artificial gauge fields. In the tight-binding language, it amounts to adding to the "bare" tunneling coefficients a phase factor proportional to the line integral of A. Despite its popularity, the PS is a rather uncontrolled approximation. Though this has been remarked from time to time in the literature [6,7], the general tendency is to use the PS as a sort of minimal coupling for tight-binding models, and this can lead to severe failures.
In a recent work [8], we have pointed out that the conditions for the applicability of the PS are explicitly violated in the Haldane model and in any other model where the vector potential varies on the same scale of the underlying lattice. Nonetheless, we have shown that the general structure of the Haldane model is in fact preserved, as it is a direct consequence of the symmetries of the system, and no additional assumptions are required. In addition, we have shown that the values of the tunneling coefficients can be obtained from simple closed expressions in terms of gauge invariant, measurable properties of the spectrum (namely, the gap at the Dirac point and the bandwidths). These formulas evidence that the phase acquired by the next-to-nearest tunneling amplitude is quantitatively different from that predicted by the PS, and it also presents a pronounced dependence on the intensity of the underlying lattice potential. Moreover, even the tunneling amplitudes turn out to be dependent on the intensity of A, violating the hypotheses behind the PS. These results have been also checked against ab-initio calculations by means of the maximally localized Wannier functions (MLWFs) [9], which are also helpful in understanding the origin of the breakdown of the PS.

I will present a pedagogical introduction to the Peierls substitution, the role of the MLWFs in the construction of tight-binding models for ultracold atoms in optical lattices, and their implications in the derivation of the Haldane model.

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