
Analytical results for the distribution of shortest path lengths in random networks

Eytan Katzav^{*1}, Ofer Biham¹, Reimer Kuehn², and Mor Nitzan¹

¹Racah Institute of Physics, the Hebrew University of Jerusalem – Israel

²King's College London – United Kingdom

Abstract

The increasing interest in network research in recent years is motivated by the realization that a large variety of systems and processes which involve interacting objects can be described by network models. In these models, the objects are represented by nodes and the interactions are expressed by edges. The interactions between non-adjacent pairs of nodes are facilitated by paths going through intermediate nodes and edges. The shortest paths between such pairs are of particular importance because they provide the strongest interactions and fastest response. Therefore, the distribution of shortest path lengths (**DSPL**) is of great relevance to many dynamical processes taking place on networks such as diffusive processes, first passage processes, traffic flow, communication and epidemic spreading.

We argue that the DSPL is a natural quantity by which dynamical processes on networks should be formulated. In particular, it incorporates the statistical symmetries of the network in the dynamical equations, in analogy to the reduction of a partial differential equation in Euclidean space to an ordinary differential equation for the radial component, in the presence of a spherical symmetry, and hence of fundamental importance.

While the average of the DSPL has been studied extensively, the analytical calculation of the entire distribution has remained an open problem. In this presentation a novel *analytical* approach for calculating the DSPL in random networks will be discussed. This approach is based on the cavity method, and applies to a large family of network types, which includes Erdos-Renyi networks [1], regular graphs and more generally, configuration model networks [2]. The results are found to be in agreement with numerical simulations for a broad range of networks, sizes and connectivities.

E. Katzav, M. Nitzan, D. ben-Avraham, P. L. Krapivsky, R. Kuhn, N. Ross and O. Biham, Analytical results for the distribution of shortest path lengths in random networks, EPL **111**, 26006 (2015).

M. Nitzan, E. Katzav, R. Kuhn and O. Biham, Distance distribution in configuration model networks, Phys. Rev. E **93**, 062309 (2016).

^{*}Speaker