Explicit computation of Reynolds stresses through statistical mechanics approaches

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Abstract

It is extremely uncommon to be able to predict analytically, from first principle, the velocity profile of turbulent flows and being able to compare it to observations. The self organization of barotropic turbulence into large scale stable structures evolving much slower than the typical time of eddies offers a unique opportunity for such an achievement through non-equilibrium statistical mechanics approaches. Jupiter's jets are a unique example, offering both this simple dynamical setup and a large amount of observations. We model the dynamics of Jupiter's jets by averaging the dynamics of eddies, in a barotropic beta-plane model, and explicitly predicting the balance between Reynolds' stresses and dissipation, thus predicting the average velocity profile explicitly [2].

In order to obtain this result, we adopt a non-equilibrium statistical mechanics approach. We consider a relevant limit for Jupiter troposphere, of a time scale separation between inertial dynamics on one hand, and stochastic forcing and dissipation on the other hand. A kinetic theory based on a quasilinear approach has been proven to be self consistent and exact in this inertial limit [1], however not giving fully explicit results. Starting from this kinetic theory and assuming further that the forcing acts on scale much smaller than the jet scale, we obtain a very simple explicit relation between the Reynolds stress, the energy injection rate, and the average velocity shear, valid far from jet edges (average velocity extrema) [2]. A specific asymptotic expansion close to jet edges unravel an asymmetry between eastward and westward, velocity extrema. We recover Jupiter's jet specificities: a cusp on eastward jets and a smooth parabola on westward jets.

While obtaining such analytic theory of barotropic jet, in accordance with Jupiter's observation is extremely encouraging, it may not be sufficient to describe all the processes at hand on Jupiter's troposphere. We discuss possible future generalization to more comprehensive model, including for instance baroclinic instabilities.

F. BOUCHET, C. NARDINI, and T. TANGARIFE, 2013, Kinetic theory of jet dynamics in the stochastic barotropic and 2D Navier-Stokes equations, J. Stat. Phys., Volume 153, Issue 4, pp 572-625

E. WOILLEZ, and F. BOUCHET, 2016, Explicit computation of Reynolds stresses and velocity profile for Jupiter turbulent jets, to be available soon on ArXiv and Hal.

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