Maximum Entropy Analysis of Potential-Driven Flow and Transportation Networks

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Abstract

The concept of a "flow network" – a set of nodes connected by flow paths – unites many different disciplines, including electrical, communications, pipe flow, fluid flow, ground and air transportation, chemical reaction, ecological, epidemiological and human systems. Traditionally, flow networks have been analysed by conservation (Kirchhoff's) laws and (in some systems) by network mappings (e.g. Tellegen's theorem), and more recently by dynamical simulation and optimisation methods. A less well explored approach, however, is the use of Jaynes' maximum entropy (MaxEnt) method, in which an entropy - defined over the total uncertainty in the network - is maximised subject to constraints, to infer the state of the network. There is a well-established literature on the use of MaxEnt methods for the analysis of network structures (graph ensembles) subject to various configurational constraints, but mostly without consideration of flows or potentials on the network.
We present a generalised MaxEnt framework to infer the state of a flow network, subject to "observable" constraints on expectations of various parameters, "physical" constraints such as conservation laws and frictional properties, and "graphical" constraints arising from uncertainty in the network structure itself. The method invokes an entropy defined over all uncertainties within the system, including all flow and potential variables and any other unknowns. The analysis also requires new numerical methods for the iterative solution of systems with nonlinear constraints. The method is demonstrated by application to several example systems, including: (i) a 1140-pipe urban water distribution network in Torrens, Australian Capital Territory, subject to nonlinear frictional constraints; (ii) a 327-node urban electrical power distribution system in Campbell, Australian Capital Territory, including distributed power sources; and (iii) several simple transportation networks subject to conservation laws and routing or cost minimisation constraints.

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