# TOY MODEL OF GEOMETRICAL COMPETITION IN NEEDLE CRYSTAL GROWTH

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**Abstract**

We present a simple dynamical model which is supposed to catch the main properties of a set of needle crystals (like, e.g., diamond needles) growing from a flat substrate. We consider needles which start growing at time zero in random directions from a set of randomly positioned seeds. Any collision of such two growing needles implies a tip of one needle hitting the body of another one. Our assumption is that on such a collision the needle whose tip hits another one stops growing ('dies'), while another one continues to grow unperturbed.
We discuss properties of this model both for an unlimited uniform initial distribution of seeds (i.e., when one has an unlimited line or plane where seeds are distributed with fixed density), and a seed distribution with limited support (i.e., all seeds are located within a given interval or half-line).

In the unlimited case we were able to find

- scaling behavior of the density of growing needles and of their angle distribution as a function of time in both (1+1)D and (2+1)D;

- the exact upper and lower bounds on the needle angle/density distribution as a function of time in (1+1)D;

- asymptotic solution of a Boltzmann equation in (1+1)D, which contradicts these bounds showing that Boltzmann approximation does not works, as typical for one-dimensional systems.

For the case with final support in (1+1)D

- given that there was a finite initial number of seeds, we calculate the full probability distribution of the number of needles surviving at infinite time;
- given that one half-line is filled with a fixed density of seeds, and another half-line is empty, we estimate the average number of needles infiltrating the originally empty half-plane up to a given time.

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