Breakdown of the Bose-Einstein condensation induced by long-range interactions within the Hartree-Fock approximation

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We consider a Bose gas with two-body interactions $V(r) = \gamma^3 v(\gamma r)$ where $v(x)$ is a given repulsive and integrable potential, while γ is a positive parameter which controls the range of the interactions and their amplitude at a given distance. Previously, within the Kac scaling, it has been proved in the literature that, in the limit $\gamma \to 0$, the gas still undergoes a Bose-Einstein condensation. This can be easily understood by noticing that for $\gamma = 0$, the particles feel a uniform potential which only shifts their kinetic energies by the constant $a\rho$, where ρ is the particle density and a is the fixed spatial integral of $V(r)$. For non-zero values of γ, that simple picture is no longer valid and the existence of a condensate is questionable. In fact, using the Hartree-Fock approximation, we find that the condensate is destroyed by the repulsive interactions when they are sufficiently long-ranged. More precisely, we show that, for γ sufficiently small but finite, the off-diagonal part of the one-body density matrix always vanishes at large distances. Our analysis sheds light on the coupling between critical correlations and long-range interactions, which might lead to the breakdown of off-diagonal long-range order. The exact status of that breakdown beyond the Hartree-Fock approximation itself remains an open question.