Critical behaviour on complex networks: inhomogeneous mean-field vs Lee-Yang-Fisher formalism

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We study the critical behavior of spin models on a scale-free network with a power-law node-degree probability distribution decay $P(k) \sim k^{-\lambda}$, $k >> 1$ and on a complete graph. To this end, we apply traditional inhomogeneous mean filed approach as well as we use the method of partition function zeros analysis in the complex temperature and magnetic field plane (Lee-Yang-Fisher formalism). The last method, to our knowledge, has not been used so far for scale-free networks.

For the Potts model on a scale-free network in terms of an inhomogeneous mean-field approach we find the set of critical amplitude ratios and scaling functions, which depend on the probability distribution decay exponent λ in the region $3 < \lambda < 5$. Moreover, we observe the non-typical behaviour for the heat capacity jump δc_H [1]: for the Ising model on a scale-free network δc_H is λ-dependent even at $\lambda > 5$, while all critical exponents are λ-independent and correspond on the values predicted by mean field theory.

We show that angles that characterize partition function zeros location in the complex plane attain λ-dependency for $3 < \lambda < 5$ too. In particular, we derive the angle at which the Fisher zeros impact onto the real temperature axis. Our analysis of the Lee-Yang zeros reveals a difference in their behaviour for the Ising model on a complete graph and on an annealed scale-free network. Whereas in the former case the zeros are purely imaginary, they have a non zero real part in the latter case, so that the celebrated Lee-Yang circle theorem is violated [2,3].

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